DAC-003-1014008 Seat No. ______

B. Sc. (Sem. IV) Examination

April - 2022

Mathematics : Paper - IV (A)

(Linear Algebra & Differential Geometry)

(Old Course)

Faculty Code : 003
Subject Code : 1014008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions.
(2) Figures to the right side indicate full marks of question.

1 (a) Answer the following questions briefly : **4**

(1) Define binary operation

(2) Define Linear combination

(3) Define Linear dependence

(4) Define Improper subspace

(b) Show that subset $\{(1,0,1),(1,1,0),(-1,0,-1)\}$ of R^3 is Linear dependence. **2**

(c) If W_1 and W_2 are sub spaces of vector space V then show that W_1+W_2 is also sub space of V. **3**

(d) $V=\{(x,y)/x>0,\ y>0,x,y\in R\}$, for $\alpha\in R$ and $(a,b),(c,d)\in V$, $(a,b)+(c,d)=(ac,bd)$ and $\alpha(a,b)=(a^{\alpha},b^{\alpha})$ then check whether V is vecor space or not. **5**

2 (a) Answer the following questions briefly : 4

(1) Define sum of two sub space

(2) Define Linear span of a set

(3) Define Linear independence

(4) Define subspace

(b) Prove that zero vector is unique in a vector space V. 2

(c) Prove that intersection of any two subspaces W_1 and W_2 of a vector space V is also a subspace. 3

(d) If vector $v_k \, (1 \le k \le n)$ of set $\{v_1, v_2, ..., v_n\}$ is linear combination of remaining vectors $v_1, v_2, ..., v_{k-1}, v_{k+1}, ... v_n$ then prove 5

$SP\{v_1, v_2, ..., v_n\} = SP\{v_1, v_2, ..., v_{k-1}, v_{k+1}, ..., v_n\}$.

3 (a) Answer the following questions briefly : 4

(1) Define Base

(2) Define Dimension of vector space

(3) What is the dimension of $P_n(R)$?

(4) Write standard base of R^3.

(b) Show that the set $\{(1,0,0),(1,0,1)\}$ of R^3 is not a base of R^3. 2

(c) Extend set $\{(2,0,0),(1,1,1)\}$ of vector space R^3 to form a base of R^3. **3**

(d) In usual notation prove **5**

$$\dim(W_1+W_2)=\dim W_1+\dim W_2-\dim(W_1\cap W_2)$$

4 (a) Answer the following questions briefly : **4**

(1) Define finite dimensional vector space

(2) Define polynomial space $P_n(R)$

(3) Write standard base of R^4

(4) Write standard base of $P_n(R)$.

(b) Show that the set $\{(2,0,1),(1,1,1)\}$ of R^3 is not a base of R^3. **2**

(c) If *W* is a sub space of finite dimensional vector space *V*. Then show that **3**

(1) $\dim W \le \dim V$

(2) $\dim W = \dim V \Rightarrow W = V$

(d) Prove that set $A=\{(1,2,1),(2,1,0),(1,-1,2)\}$ forms a base of R^3. Find co-ordinates of $(1,1,-1)$ with respect to this base. **5**

5 (a) Answer the following questions briefly : **4**

(1) Define Linear operator

(2) Define Linear functional

(3) Define Rank of Linear transformation

(4) Define Algebra

(b) Prove that $T:U \to V, T(u) = \theta', \forall u \in U$ is a linear transformation. Where θ' is zero vector of V. **2**

(c) Prove that composition of two linear transformations is again a linear transformation. **3**

(d) Prove that a linear transformation $T:U \to V$ is one-one iff $N_T = \{\theta\}$. **5**

6 (a) Answer the following questions briefly : **4**

(1) Define Linear Transformation

(2) Define Zero Linear Transformation

(3) Define Kernel of Linear transformation

(4) Define Nullity of Linear transformation

(b) Show that $T:R^2 \to R^2; T(x,y) = (x+1, y+2); \forall (x,y) \in R^2$ is not a linear transformation. **2**

(c) If $T:V \to V$ is any linear transformation such that $T^2 - T + I = 0$ then prove T is non-singular. **3**

(d) State and prove Rank-Nullity theorem. **5**

7 (a) Answer the following questions briefly : **4**

(1) Define dual of a vector space

(2) If $\dim U = m$ and $\dim V = n$ then $\dim \{L(U, V)\} = ...$

(3) Define adjoint of a linear transformation.

(4) Define Eigen value of a linear transformation.

(b) For linear transformation **2**

$T : R^2 \to R^2; T(x, y) = (2x - y, x + 2y)$. Find $[T; B]$ where B is standard base.

(c) If $T : V \to V$ is linear transformation and B is base of V then prove that λ is an eigen value of T iff **3**

$\det\left(\left[\left(T - \lambda I_n\right), B\right]\right) = 0$

(d) If $T : R^2 \to R^2; T(x, y) = (x, -y); \forall (x, y) \in R^2$ and **5**

$B_1 = \{(1,1), (1,0)\}, B_2 = \{(2,3), (4,5)\}$. Then find $[T : B_1, B_2]$.

8 (a) Answer the following questions briefly : **4**

(1) Define Eigen vector of a linear transformation

(2) Define Eigen base of a linear transformation

(3) Define diagonalization of a linear transformation

(4) If dim U = 4 and dim V = 2 then dim$\{L(U, V)\}$ = ...

(b) For linear transformation $T : R^2 \to R^2$; **2**

$T(x, y) = (x - 3y, 3x + 2y)$, Find $[T; B]$ where B is standard base.

(c) Find Eigen values of linear transformation $T : R^3 \to R^3$, **3**

$T(x, y, z) = (-2y - 2z, -2x - 3y - 2z, 3x + 6y + 5z)$, $\forall (x, y, z) \in R^3$

by considering standard base of R^3.

(d) If $T : R^3 \to R^2; T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$; **5**

$\forall (x, y, z) \in R^3$ and $B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$,

$B_2 = \{(1,3), (1,4)\}$. Then find $[T : B_1, B_2]$.

9 (a) Answer the following questions briefly : **4**

(1) Define point of inflexion

(2) Define node

(3) Define singular point of a curve

(4) Define asymptotes of a curve

(b) Find the radius of curvature of curve $s = 4a \sin \Psi$. **2**

(c) Find the radius of curvature of curve **3**

$x^2 + 2xy + 2y^2 - 4x = 0$ at origin.

(d) Obtain the formula for radius of curvature for Cartesian curves. **5**

10 (a) Answer the following questions briefly : **4**

(1) Define Oblique asymptotes

(2) Define multiple points

(3) Define cusp

(4) Define double point

(b) Prove that $y = \log x$ is convex upwards everywhere. **2**

(c) Find the asymptotes parallel to co-ordinate axis for the curve $x^2 y^2 = 4x^2 + 9y^2$. **3**

(d) Show that radius of curvature at any point on the cardiod $r = a(1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2\,ar}$. **5**
